Lesson 26. Applications of Optimization with Equality Constraints

Example 1. Eli Orchid manufactures its newest pharamceutical product, Med-X, using its two patented processes. Process 1 costs \$1,000 per batch, and Process 2 costs \$250 per batch. These two processes work in tandem: if Eli Orchid uses x_1 batches of Process 1 and x_2 batches of Process 2, it produces $10\sqrt{x_1 x_2}$ liters of Med-X. Eli Orchid wants to find the least costly way of producing 100 liters of Med-X.

a. Using the variables x_1 and x_2 defined above, write cost as a function of x_1 and x_2: $c(x_1, x_2) = \ldots$
b. Using the variables x_1 and x_2 defined above, write an equality constraint that represents that Eli Orchid must produce 100 liters of Med-X.
c. Find the local optima of the cost function c you wrote in part a, subject to the equality constraint you wrote in part b.
d. How many batches of Process 1 and Process 2 should Eli Orchid use? What is the corresponding cost?

Example 2. Suppose that you are interested in dividing your savings between three mutual funds with expected returns of 10%, 10% and 15%, respectively. You want to minimize risk while achieving an expected return of 12%. To measure risk, use the *variance* of the return on investment: when a fraction x of your savings is invested in Fund 1, y in Fund 2, and z in Fund 3, the variance of the return has been calculated to be

$$v(x, y, z) = 400x^2 + 800y^2 + 200xy + 1600z^2 + 400yz$$

a. Consider the equality constraints below. Why do these constraints make sense for this problem?

$$1.10x + 1.10y + 1.15z = 1.12 \tag{1}$$

$$x + y + z = 1 \tag{2}$$

b. Find the local optima of the variance v, subject to the equality constraints given in part a.

c. How much should you invest in the three mutual funds?