SM286A – Mathematics for Economics Fall 2015
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Lesson 27. Review

Set A

Problem 1. Suppose you have a system of 4 linear equations with 5 variables: x_1, x_2, x_3, x_4, and x_5. You form the augmented matrix of this system, and find its RREF:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Based on this, give 2 solutions to the original system of equations.

Problem 2. Find the RREF of

$$A = \begin{bmatrix} -2 & 0 & 1 \\ 2 & -1 & 0 \\ -1 & 2 & -1 \end{bmatrix}$$

What is the rank of A? Is A invertible?

Problem 3. In a two-industry economy, it is known that industry 1 uses 5 cents of its own product and 40 cents of industry 2's product to produce a dollar's worth of its own product. Industry 2 uses none of its own product but uses 20 cents of industry 1's product in producing a dollar's worth of its own product. The final demands for industry 1's product and industry 2's product are is \$2,000, and \$1,000, respectively.

Write the input matrix and the Leontief matrix for this economy. Find the solution output levels using Cramer's rule.

Problem 4. Draw the phase diagram for the differential equation $y' = (y-2)^2 - 1$. What are the equilibrium points? Sketch the graph of $y(t)$ when $y(0) = 0$, $y(0) = 2$, and $y(0) = 4$. What is $\lim_{t\to\infty} y(t)$ when $y(0) = 0$?

Problem 5. Consider the following model of a market with a single product in discrete time. The model variables are

P_t = unit price in period t Q_{dt} = quantity demanded in period t Q_{st} = quantity supplied in period t

The model equations are

$$\begin{aligned} Q_{dt} &= Q_{st} \\ Q_{dt} &= 5 - 2P_t \\ Q_{st} &= -1 + 4P_{t-1} \end{aligned}$$

Combine these equations to find a difference equation that governs the price in each period. Find the solution to this difference equation. What happens to the price of the product in the long run?

Set B

Problem 1. Solve the differential equation $y' + 4ty = 4t$ with the initial condition $y(0) = 2$.

Problem 2. Write the Solow growth model equation when the marginal propensity to save is 2, the labor growth rate is 4, and the production function is $f(K, L) = K^{1/4}L^{3/4}$. What does a solution to the Solow growth model equation represent?

Problem 3. Solve the difference equation $y_{t+1} + 3y_t = 4$, with initial condition $y_0 = 2$. Is y_t oscillatory or nonoscillatory? Is y_t convergent or divergent?

Problem 4. Find $|A|$, where

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 9 & 1 & 3 & 0 \\ 9 & 2 & 2 & 0 \\ 5 & 0 & 0 & 3 \end{bmatrix}$$

Problem 5. Consider the following system of equations:

$$\begin{aligned} x \quad\quad + \quad z &= 2 \\ 4y - 2z &= 8 \\ 4x - \quad y + 5z &= 4 \end{aligned}$$

You are given that

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & -2 \\ 4 & -1 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 9 & -1/2 & -2 \\ -4 & 1/2 & 1 \\ -8 & 1/2 & 2 \end{bmatrix}$$

Use this information to solve for x, y, and z.