B4.

$$\begin{vmatrix} 1 & 0 & 1 & 2 \\ 9 & 1 & 3 & 0 \\ 9 & 2 & 2 & 0 \\ 5 & 0 & 0 & 3 \end{vmatrix} = -5 \begin{vmatrix} 0 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 2 & 0 \end{vmatrix} + 3 \begin{vmatrix} 1 & 0 & 1 \\ 9 & 1 & 3 \\ 9 & 2 & 2 \end{vmatrix}$$

$$= -5\left[2\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix}\right] + 3\left[\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 9 & 1 \\ 9 & 2 \end{vmatrix}\right]$$

$$= -10(-4) + 3\left[-4+9\right] = 40 + 3(5) = 55$$

B5. The system of equations can be written as

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & -2 \\ 4 & -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

and so

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & -2 \\ 4 & -1 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 & -1/2 & -2 \\ -4 & 1/2 & 1 \\ -8 & 1/2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 18 - 4 - 8 \\ -8 + 4 + 4 \\ -16 + 4 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -4 \end{bmatrix}$$

A1. Solutions by solving for leading variables x_1, x_3, x_4 in terms of free variables:

$$x_1 = 2 - 2x_2 - 3x_5$$
$$x_2 = x_2$$
$$x_3 = 4 + x_5$$
$$x_4 = 3 + 2x_5$$
$$x_5 = x_5$$

Plug in arbitrary values for free variables to get example solutions:

ex. $x_2 = 0, x_5 = 0 \Rightarrow (2, 0, 4, 3, 0)$

$x_2 = 1, x_5 = 0 \Rightarrow (0, 1, 4, 3, 0)$

A2.

$$\begin{bmatrix} -2 & 0 & 1 \\ 2 & -1 & 0 \\ -1 & 2 & -1 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_1} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 2 & -1 & 0 \\ -1 & 2 & -1 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix} \xrightarrow{R_3 + R_1} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & -1 & 1 \\ 0 & 2 & -\frac{3}{2} \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 \\ 0 & 2 & -\frac{3}{2} \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \xrightarrow{2R_3} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 + \frac{1}{2}R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rank(A) = 3 $\Rightarrow$ A is invertible

A3.

input matrix $A = \begin{bmatrix} 0.05 & 0.20 \\ 0.40 & 0 \end{bmatrix}$ Leontief matrix $I-A = \begin{bmatrix} 0.95 & -0.20 \\ -0.40 & 1 \end{bmatrix}$

equation: $(I-A)x = d$ or $\begin{bmatrix} 0.95 & -0.20 \\ -0.40 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \end{bmatrix}$

Cramer's rule: $x_1 = \dfrac{\begin{vmatrix} 2000 & -0.20 \\ 1000 & 1 \end{vmatrix}}{\begin{vmatrix} 0.95 & -0.20 \\ -0.40 & 1 \end{vmatrix}} = \dfrac{2200}{0.87} = 2528.74$

$x_2 = \dfrac{\begin{vmatrix} 0.95 & 2000 \\ -0.40 & 1000 \end{vmatrix}}{\begin{vmatrix} 0.95 & -0.20 \\ -0.40 & 1 \end{vmatrix}} = \dfrac{1750}{0.87} = 2011.49.$

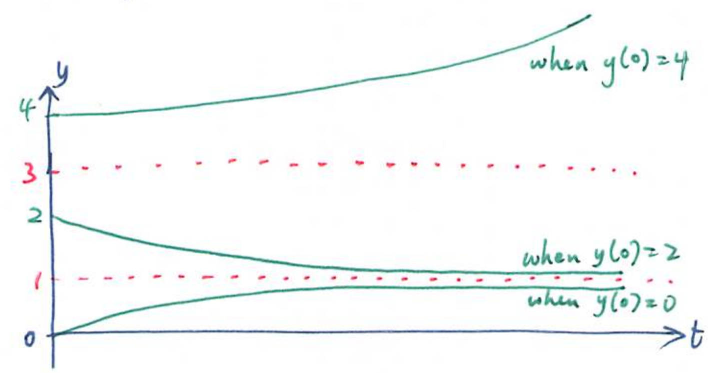
B1. $u = 4t \qquad w = 4t$

$\int u\,dt = \int 4t\,dt = 2t^2 \qquad \int w e^{\int u\,dt}\,dt = \int 4te^{2t^2}\,dt = e^{2t^2}$

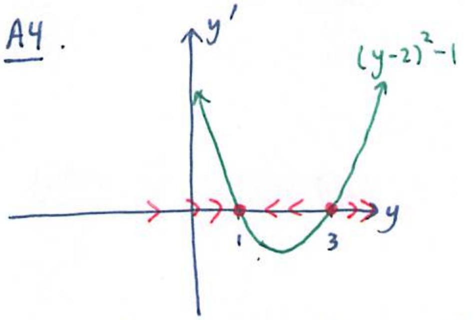
$\Rightarrow y(t) = e^{-2t^2}\left(A + e^{2t^2}\right) = Ae^{-2t^2} + 1$

Initial condition: $y(0) = A + 1 = 2 \Rightarrow A = 1$

Solution: $y(t) = e^{-2t^2} + 1.$



A4.



eq. points: $y=1$ - dynamically stable
$y=3$ - dynamically unstable

By the graph above (and the phase diagram),

$\lim_{t\to\infty} y(t) = 1$ when $y(0) = 0$.

B2. Solow growth model: $\quad k' = \overset{2}{s}\,\phi(k) - \overset{4}{\lambda} k \qquad k = \text{capital-to-labor} = \frac{K}{L}$

$$\phi(k) = \frac{f(K,L)}{L} = \frac{K^{1/4}L^{3/4}}{L} = \left(\frac{K}{L}\right)^{1/4} = k^{1/4} \quad \Rightarrow \quad \text{model eq: } k' = 2k^{1/4} - 4k.$$

Solve Solow growth model $\Rightarrow$ get $k(t)$ = capital-to-labor over time.

B3. $a = 3,\ c = 4 \quad \Rightarrow \quad y_t = \left(2 - \frac{4}{1+3}\right)(-3)^t + \frac{4}{1+3} = (-3)^t + 1$

oscillatory, since $b = -a < 0$
divergent, since $|b| = |-a| > 1$.

A5. $5 - 2P_t = -1 + 4P_{t-1} \Rightarrow 2P_t + 4P_{t-1} = 6 \Rightarrow P_t + \overset{a}{2}P_{t-1} = \overset{c}{3}$

$$\Rightarrow P_t = \left(P_0 - \frac{3}{1+2}\right)(-2)^t + \frac{3}{1+2} = (P_0 - 1)\underline{(-2)^t} + 1.$$

Price oscillates and diverges in the long run b/c of