Name: Feedback

SM339 · Applied Statistics Spring 2024 · Uhan

# Exam 1 – Part 1 – 2/13/2024

## Instructions

- This part is worth 60 points total. The exam (both parts) is worth 100 points total.
- You have 50 minutes to complete this part of the exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may refer to notes that you have handwritten, not to exceed one side of an 8.5" × 11" piece of paper.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

| Problem | Weight | Score |
|---|---|---|
| 1a | 0.4 | |
| 1b | 0.4 | |
| 2a | 0.4 | |
| 2b | 0.4 | |
| 2c | 0.4 | |
| 2d | 0.4 | |
| 2e | 0.4 | |
| 3a | 0.4 | |
| 3b | 0.4 | |
| 3c | 0.4 | |
| 3d | 0.4 | |
| 3e | 0.4 | |
| 4a | 0.4 | |
| 4b | 0.4 | |
| 5 | 0.4 | |
| Total | | / 60 |

**Problem 0.** Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Signature:

**Problem 1.** You have just been hired as an operations research analyst at MedTech Diagnostics, a medical testing company. They are interested in the average patient waiting time at their Simplexville location over the past year. You take a random sample of 23 patients at the Simplexville location over the past year and find that the sample mean patient waiting time is 16.74 minutes with a sample variance of 23.9.

a. Construct a 95% confidence interval for the mean patient waiting time at the Simplexville location. Provide your answer to 3 decimal places.

   You may find some of the following R output helpful:

| Code | Output |
|---|---|
| `qnorm(1 - 0.05/2, mean = 0, sd = 1)` | 1.9600 |
| `qnorm(1 - 0.95/2, mean = 0, sd = 1)` | 0.0627 |
| `qt(1 - 0.05/2, df = 22)` | 2.0739 |
| `qt(1 - 0.95/2, df = 22)` | 0.0634 |

   See the example in Lesson 3 Part 2 and Problems 1 and 2 in the Lesson 3 Part 2 Exercises for similar problems.

   - Note that the sample variance is given, and that the population variance is not given. See Lesson 3 Part 1 for details on the difference between computing CI for population mean when the population variance is known versus not known.
   - Remember that the sample standard deviation is the square root of the sample variance.
   - Note that $t_{\alpha/2,n-1}$ is given by `qt(1 - alpha / 2, df = n - 1)` in R.
   - A confidence interval is just like any other interval: it should be written in the form (lower value, upper value).

b. You write in your report that you are "95% confident" that the interval you found in part a contains the true mean patient waiting time. Briefly explain what this means.

   See page 3 of Lesson 3 Part 1 for an explanation of what it means to be "95% confident."

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**Problem 2.** Continuing with the setting from Problem 1...

You learned from your predecessor at MedTech Diagnostics that two years ago, the average patient waiting time at the Simplexville location was 16 minutes. You're interested in whether the average patient waiting time at the Simplexville location over the past year has increased in a statistically significant way.

You use the same data as before: based on a random sample of 23 patients at the Simplexville location over the past year, the sample mean patient waiting time is 16.74 minutes with a sample variance of 23.9.

Perform a $t$-test for one population mean by answering the following prompts.

a. Let $\mu$ be the mean patient waiting time at the Simplexville location over the past year. State the null and alternative hypotheses.

   Note that you are testing whether the average patient waiting time has increased significantly over the past year. See Example 2 in the Lesson 4 Part 1 for a similar problem.

b. Calculate the test statistic. Provide your answer to 3 decimal places.

   See Example 2 in the Lesson 4 Part 1 for a similar problem. See Lesson 4 Part 1 for details on how to compute the test statistic. Again, remember that the sample standard deviation is the square root of the sample variance.

c. Suppose the $p$-value is 0.2378. Using a significance level of 0.10, do you reject or fail to reject the null hypothesis? Briefly explain.

   See Lesson 4 Part 1 for details on how to use the $p$-value in the $t$-test for one population mean. Note that the problem only asks if you reject or fail to reject the null hypothesis, and to explain why. You do not (and should not) write the resulting conclusion for this part.

d. Based on your answer to part c, state your conclusion about the average patient waiting time at the Simplexville location.

   See Lesson 4 Part 1 for details on how to write a conclusion for the $t$-test for one population mean.

   Note that your conclusion should be framed in terms of the alternative hypothesis. In this case, the alternative hypothesis is that the population mean is greater than $\mu$, so you should state one of the following:

   - We see evidence that the population mean is greater than $\mu$.
   - We do not see evidence that the population mean is greater than $\mu$.

   As usual, you should rephrase the underlined parts to match the context of the problem.

e. Suppose we fail to reject the null hypothesis when in fact it is false. What type of error have we committed? No explanation necessary.

   See Lesson 4 Part 1 for details on Type I and Type II errors.

**Problem 3.** You are working with data for 36 breakfast cereals. Your data consists of two variables: *Calories* per serving and grams of *Fiber* per serving. You are interested in predicting *Calories* based on *Fiber*. With this data, you fit a simple linear regression model.

Below is output from `summary()` for your model:

```
Call:
lm(formula = Calories ~ Fiber, data = Cereal)

Residuals:
    Min      1Q  Median      3Q     Max
-17.363  -7.363  -4.005   1.413  55.801

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 117.3635     3.7216  31.536  < 2e-16 ***
Fiber        -4.3881     0.7358  -5.964  9.6e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.72 on 34 degrees of freedom
Multiple R-squared:  0.5112,  Adjusted R-squared:  0.4969
F-statistic: 35.56 on 1 and 34 DF,  p-value: 9.603e-07
```

a. Write the fitted model. Report all coefficients to 3 decimal places.

See Example 2 in Lesson 7 Part 1 for a similar example.

Note that you can find the name of the variables in the simple linear regression model in second line of the output from `summary()` above.

b. Interpret the slope in the context of the problem. Make sure to include units in your interpretation.

See Lesson 7 Part 1 for details on how to interpret the slope of a simple linear regression model. Also see Example 2 in Lesson 7 Part 1, and STAT2 Exercises 1.19, 1.45 assigned for homework for similar problems.

Be careful with your language here. Remember that the fitted model represents how the average value of the response variable is related to the explanatory variable. Your explanation should include the words "average" or "expected".

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c. Predict the number of calories per serving for cereal that has 5 grams of fiber per serving. Provide your answer to 3 decimal places.

See STAT2 Exercise 1.21a assigned for homework for a similar problem.

d. Compute the residual for a cereal that has 3 grams of fiber and 100 calories per serving. Provide your answer to 3 decimal places.

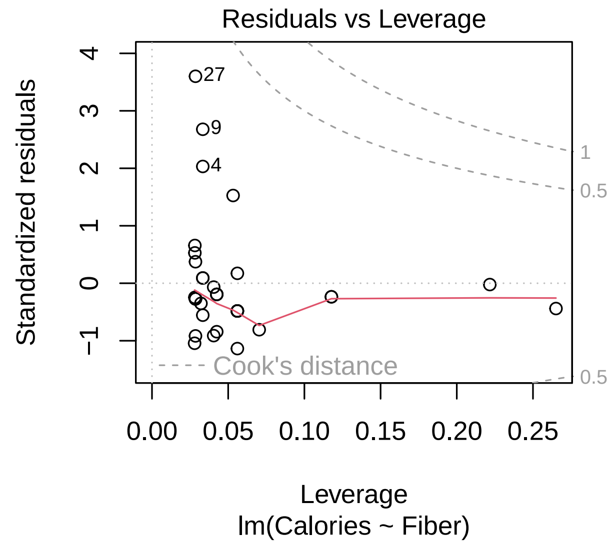
See Lesson 7 Part 1 for the definition of residual. In addition, see Example 2 in Lesson 7 Part 1 and STAT2 Exercise 1.21b assigned for homework for a similar problem.

e. What is the size of a typical error when predicting calories from fiber content?

See Lesson 7 Part 1 for more details on the size of a "typical error" and the estimated standard deviation of the error term. In addition, see Example 3 in Lesson 7 Part 1 for a similar problem.

**Problem 4.** Continuing with the setting from Problem 3...

Using R, you generate the diagnostic plot on the right for the model you fit in Problem 3.




a. Based on the rules of thumb we covered in class, circle and label the points that are classified as "very unusual" leverage points. Briefly explain your reasoning below.

See Lesson 10 for the rule of thumb we covered in class that classifies points as "very unusual" leverage points.

b. Based on the rules of thumb we covered in class, circle and label the points that are classified as "very unusual" outliers. Briefly explain your reasoning below.

See Lesson 10 for the rule of thumb we covered in class that classifies points as "very unusual" outliers.

**Problem 5.** A capacitor was charged with a 9-volt battery and then a voltmeter recorded the voltage as the capacitor was discharged. Measurements were taken every 0.02 seconds. Your data consists of two variables: *Voltage* (in volts) and *Time* (in seconds). You are interested in predicting *Voltage* based on *Time*.

After exploring the data, you decide that applying a log transformation to *Voltage* is appropriate. You fit a simple linear regression model with $log(Voltage)$ as the response variable, and *Time* as the explanatory variable (assume *log* is the natural logarithm). Your fitted model is

$$log(\widehat{Voltage}) = 2.19 - 2.06 Time$$

Use your fitted model to predict *Voltage* when $Time = 0.05$ seconds.

See Example 1 in Lesson 9 for a similar problem.

Note that the problem specifies that *log* is the natural logarithm!