Lesson 15. The Multiple Linear Regression Model – Part 1

Note. In Part 2 of this lesson, you can run the R code that generates the plots and outputs in here Part 1.

1 Overview

- We still want to study or predict the behavior of a response variable Y...
- But now, we will use multiple explanatory variables $X_1, X_2, \ldots, X_k$

2 Choosing a multiple linear regression model

- We need:
  1. One quantitative response variable
  2. Multiple explanatory variables (quantitative or categorical)
- Suppose we have n observations of k explanatory variables $(X_1, \ldots, X_k)$ and a response variable Y
- The **multiple linear regression model** is:

  - β_j describes the relationship between Y and X_j when all the other explanatory variables are held constant

3 Fitting a multiple linear regression model

- We still use **least squares regression** to estimate the best fit
- The **fitted model** (or prediction equation) is:

  - The estimated coefficient $\hat{\beta}_i$ describes the estimated average relationship between the response variable Y and the explanatory variable X_i when all the other explanatory variables are fixed
  - Interpretation:

    On average, an increase/decrease of 1 unit in the explanatory variable is associated with an increase/decrease of $|\hat{\beta}_i|$ in the response variable, holding all other explanatory variables fixed.

  The underlined parts above should be rephrased to correspond to the context of the problem

- Let y_i be the observed value of the response variable Y for observation i
- Let x_{ji} be the observed value of the explanatory variable X_j for observation i
- The predicted value of the response variable Y for observation i is:

- The **residual** of observation i is still defined as:

- The **estimated standard error of the multiple regression model** with k predictors is:

    - This is still interpreted as the size of a "typical" prediction error

4 Assessing a multiple linear regression model

- The conditions and assumptions are analogous to those in simple linear regression

Condition	**Where to check**	**What we want**
Linearity	Residuals vs. fitted values plot	Points randomly and evenly distributed above and below residual = 0 line, moving from left to right
Independence	Description of data collection	No indication that the errors influence each other
Normality	Normal Q-Q plot of residuals	Points in approximately straight line
Equal variance	Residuals vs. fitted values plot	Points span constant vertical width, moving from left to right
Randomness	Description of data collection	Data obtained using a random process, such as random sampling from a population or randomization in an experiment

Example 1. How is an NFL team's winning percentage related to its offensive and defensive performance? The dataset `NFLStandings2016` from `Stat2Data` contains the records for all NFL teams during the 2016 regular season. *WinPct* is the winning percentage, *PointsFor* is the total number of points scored, and *PointsAgainst* is the total number of points allowed.

a. What is the response variable? What are the explanatory variables?

b. Write the population-level model; that is, the model we will fit. Include the distribution of the error term.

c. We can fit the multiple regression using R with the following code:

```
fit <- lm(WinPct ~ PointsFor + PointsAgainst, data = NFLStandings2016)
summary(fit)
```

We get the following output:

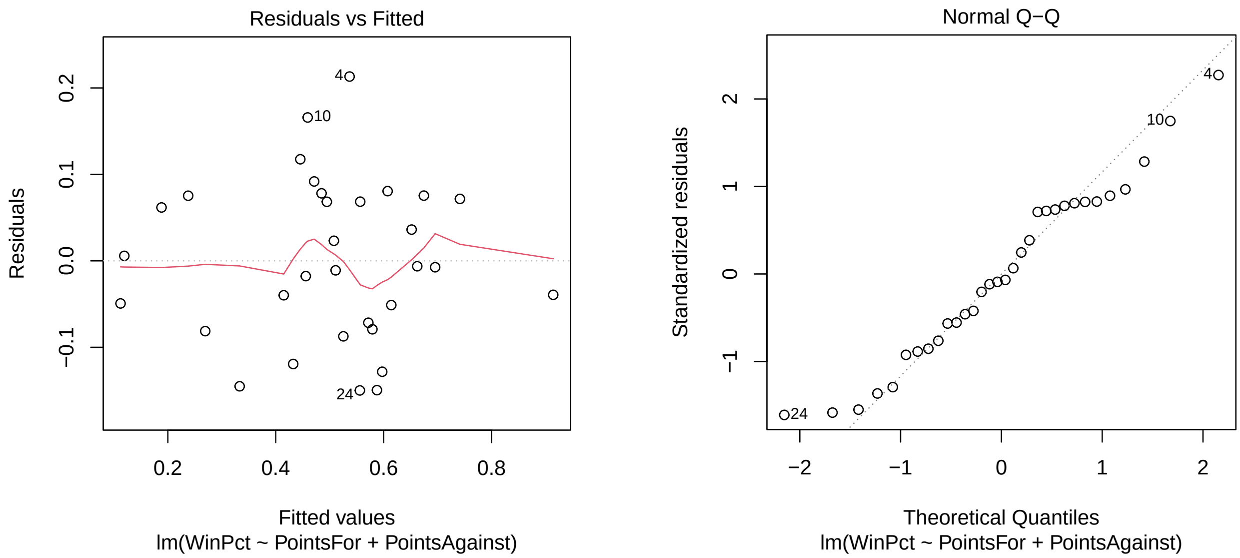
```
Call:
lm(formula = WinPct ~ PointsFor + PointsAgainst, data = NFLStandings2016)

Residuals:
      Min        1Q    Median        3Q       Max
-0.149898 -0.073482 -0.006821  0.072569  0.213189

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.7853698  0.1537422   5.108 1.88e-05 ***
PointsFor      0.0016992  0.0002628   6.466 4.48e-07 ***
PointsAgainst -0.0024816  0.0003204  -7.744 1.54e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.09653 on 29 degrees of freedom
Multiple R-squared:  0.7824,  Adjusted R-squared:  0.7674
F-statistic: 52.13 on 2 and 29 DF,  p-value: 2.495e-10
```

Write the fitted model.

d. Assess whether the conditions for multiple regression appear to be met. The code below should look familiar to you – it creates a residuals vs. fitted values plot and a Normal Q-Q plot of the residuals:

```
plot(fit, which=1)
plot(fit, which=2)
```

The output is below:



Linearity	
Independence	
Normality	
Equal variance	
Randomness	

e. Consider the Baltimore Ravens who scored 343 points while allowing 321 points during the 2016 season.

   i. What is their predicted winning percentage?

   ii. Their winning percentage was actually 0.500. What is the corresponding residual?

f. What is the estimated regression standard error?

g. Interpret the estimated coefficient of *PointsFor*.

h. What is the predicted increase in *WinPct* associated with a 7 point increase in *PointsFor* (holding *PointsAgainst* fixed)?